1 Teaching Philosophy

My teaching philosophy is simple: Mathematical intuition and logic are fundamental to developing the problem solving skills of students of all disciplines. Not all students are destined to become scientists. All students will need to be able to critically think on their feet when presented with complex problems. More than seven years of teaching experience have shown me, among other things, that it is important to engage students using real life applications. Working closely with students has helped me to get another perspective on my own research and teaching skills. I have a strong interest in developing optimal strategies for teaching mathematics to students of all backgrounds and fields of study. As my career develops, it becomes more apparent that teaching and research go hand in hand. I would like to continue to challenge students with open ended questions and pose current, real life problems to help establish foundational knowledge as well as honing research skills. It is important to me to not only stay current in the world of research, but to also develop parallel programs for students so they can learn contemporary problem solving.

There are several key techniques I use to efficiently educate students of all levels and backgrounds. They are outlined as follows.

Active Engagement of Students

The common lecture format of many mathematics courses is outdated and does not engage students in active learning.The lecture prevails as the primary method of instruction, leaving many students unengaged. The method of sponging - instructors excreting massive amounts of information for students to absorb - is not an effective way of preparing students for any career, much less those requiring scientific intuition and collegiate interaction. Memory is an imperfect thing - it is not conserved, and is constantly adapting to account for acquisition of new information. We need to engage our students in discussion with their instructor and their peers in the classroom so that their brains do not lay dormant during this optimal time of knowledge absorption. Rather than presenting a problem and solving it while students observe, I try to pose problems in different ways. While lecturing, I ask students qualitative questions about each step of an equation that I am deriving or solving. For some problems I have students come to the board and solve with the help of their peers. It is important to remember they need encouragement - above all else students are fearful of failure, and so need constant encouragement to explore new ways of learning. Other topics I will address with worksheets and/or labs completed in small groups that I can interact with individually. The worksheets have problems guiding students through the step by step derivation and analysis of problems so they get hands-on experience solving unfamiliar problems, rather than watching the instructor do so. No mathematical principle comes out of thin air - it is always motivated with a connection to our previous topics and applications.

Learning with Applications

Presenting students with real-world problems with no one definite solution is an excellent method of actively engaging students. Students in math classes often find it difficult to visualize and gain deeper understanding of concepts without concrete relations to real-world applications. I integrate mathematical visualizations into my lectures and refer students to hands-on online references to explore outside of the classroom. My assignments are never just simple plug-and-chug problems. We warm up with simple applications of theorems learned during the lecture. I then focus on guiding students step by step through a concept and eliciting insightful responses. All forms of math problems have a place in the classroom.

Promoting Student Responsibility

Students will rise to the occasion when given a task they feel responsible for. For recent courses that I have taught, I include a preliminary survey regarding students interests, both academic and not, and past mathematical experience. This allows me to adjust my material and teaching strategies to the students' needs. Incorporating elements of research into assignments helps to make students feel ownership of the problems they are addressing.

Student-Instructor Interaction

Engaging students in active classroom problem solving is not just an excellent teaching tool - it bolsters the instructor-student relationship. I pose problems during class time for my students to answer out loud or on the board. You can diminish fear, the greatest hinderance of student learning, simply being comfortable and friendly with your students, sharing your research interests and providing a safe classroom environment more conducive to learning. I take class time to work on problems with students in small groups so that I am able to speak to them and observe their work style individually. I review all of my students assignments regardless of whether I have a grader. Teaching can often hold the stress of a performance - you must stand in front of an unfamiliar classroom of relay your knowledge to a group of eager and inexperienced young minds. A good teacher can admit their mistakes humbly and then use it as a lesson for their students. I have learned more from observing students address problems than I ever could have from years of school. I adapt my material and the way I present it after listening to student concerns and input. This ensures a more rich experience for future generations of students. I make myself readily available to all of my students, and encourage them to pursue mathematics outside of the classroom. I regularly recruit my students for the Mathematical Contest in Modeling, which I discuss more below. We as teachers have the responsibility of building a reliable knowledge base for hundreds of students who will then go on to forge careers of their own.

For the Love of Learning

We must be able to adapt our teaching strategies to anticipate different students as well as changes in the world of academia. As I stated before, not all students are destined to become scientists. All students need to develop problem solving skills, and all students should share in the joy of learning. Students work harder and develop a more positive academic attitude when I relay my enthusiasm for mathematics education and research. Along with incorporating applications, I make sure to emphasize the simplicity and beauty of the underlying mathematical theory. I hope to inspire a passion for mathematics and knowledge in general to all ages and walks of life.

2 Teaching Documentation, Effectiveness and Evaluation

Computational Science Training for Undergraduates in the Mathematical Sciences (CSUMS), RPI, 2006-2008

I was the TA for and helped to obtain the grant for the NSF-funded RPI CSUMS (Computational Science Training for Undergraduates in the Mathematical Sciences) program. This is a collaborative program involving Rensselaer and Howard University. The goal of this program was to train undergraduates for research careers in computationally-intensive sciences. The course began with lectures on computational methods for solving and analyzing systems of differential equations given by several expert professors in the department. After some practice with these methods, students had to choose a professional paper on a research topic of choice. The remainder of the semester was spent meeting with research advisors in small groups developing original projects on their topics of choice. At the end of the semester, each topic group had to give an oral and written report discussing their project. Eight of the most promising students went on to funded summer research, for which I was the group discussion session organizer and advisor. They then presented their material at Howard University and at the Hudson River Undergraduate Math Conference.

CSUMS Website

The students were given access to a comprehensive research paper bank to browse for topics, found here:

CSUMS Library.

Student Projects

- Brenton Blair, *Short Term Effects of Acid Precipitation on a Three Species Aquatic Ecosystem*. Brenton collaborated with Brad Lister, Peter Kramer and myself to build a model representing the fluctuating ecosystem of an at-risk Adirondack lake. He used real data from recent experiments and limnology papers to form a system of equations describing the changing pH levels of the lake as well as the competition between species in the lake.
- Pamela Fuller and Toni Wagner, *Vaccinating Against HPV: An attempt to further the prevention of cervical cancer*. Pamela and Toni collaborated with Peter Kramer and myself to explore vaccination patterns on a sexual network to minimize the number of cervical cancer cases resulting from the Human Papillomavirus (HPV). They utilized data and observations from Merck and the Center for Disease control to build a realistic human sexual network, seed the virus and compare various vaccination strategies.

Syllabus: Attached
No Evaluations available at this time.

Math Modeling, Johns Hopkins Center for Talented Youth at Skidmore University, Summer 2006

I was the instructor for two sessions of Math Modeling at the Saratoga Springs site for the Center for Talented Youth summer program in 2006. Students aged 10 to 14 learned various mathematical modeling principles and what they could do with a career in mathematics research. At the end of the course, each student gave a short written and oral report on a research topic of choice. Topics included Keynsian Economics, Chemical Kinetics and mathematically modeling tsunami dynamics.
Syllabus: Attached
No Evaluations available at this time.

The Mathematical Contest in Modeling, RPI and CIMS, 2007-Present

While a graduate student at RPI, I developed and ran problem training sessions for the Mathematical Contest in Modeling. I continue to do so at Courant. This is a once yearly four day contest in which small groups of students perform research and use mathematical modeling to present their solutions to real world problems. During practice problem sessions, students are posed with past Mathematical Contest in Modeling problems and other real world problems. You can read the complete details in my Teaching Statement. The students this year have been so enthusiastic, that they requested we have problem sessions year round rather as just a precursor for the contest. After the contest this year, we will form the Mathematical Modeling Club, and will meet weekly to brainstorm solutions for current real-world math problems. I hope to slowly turn the club responsibilities over to the students so that they can continue to have the Mathematical Modeling Club after I have gone on to another job.

Multivariable Calculus, CIMS, Fall 2010, Spring 2011

Syllabus: Attached
Sample Assignments: Attached
Evaluations

- "Ms. Rogers is an excellent instructor who I find extremely skilled in her teaching. She is able to explain the course materials clearly and offer adequate examples to further aid in my understanding."

- “She is a good professor. She helped me understand the material well.”
- “Challenging, but not too much so. Very clear explanations.”

Calculus 2, CIMS, Fall 2012

Syllabus: Attached
Sample Assignments: Attached
Evaluations

- “Great teacher, clearly cares about the topics. Class is conducted in the right way (i.e. learning is more important than grades).”
- “The teacher is great, but the material she teaches is hard.” (I find this one amusing)

Introduction to Mathematical Modeling, CIMS, Spring 2013

I am very excited to teach this course. This will be the first course I have total control over creating. I am currently working on formalizing assignments and making the final edits to the syllabus. Please contact me for materials.
No Evaluations available at this time.

Calculus for the Natural and Physical Sciences

I have worked on developing an applied Calculus course for students pursuing careers in the natural and physical sciences. This is merely a proposal and has not been implemented, though I would love the opportunity to do so! The presentation for proposal of this course is attached.
Presentation: Attached

3 Contributions

Ready... Set... Calculus, 2005

Ready... Set... Calculus is a text developed by professors from Rensselaer Polytechnic Institute to help incoming freshman assess and enhance their pre-college Calculus skills. I helped to edit, create and generate solutions for problems found in the text. Reference: Harry McLaughlin, mclauh@rpi.edu. Online Text

Illustrative Math Project, 2012

I recently completed my training to edit and submit new applied math problems to the Illustrative Mathematics Project, which is a wonderful project involving mathematics teachers from all backgrounds and academic levels. Contributors can browse, submit and edit math problems addressing the Common Core State Standards in creative, applied ways. I plan on developing a branch of this project for collegiate-level mathematics. Reference: Selin Kalaycioglu, kalaycioglu@cims.nyu.edu. Illustrative Mathematics Website

RPI Department Head search, 2009

I was asked to take part in the search for a new mathematics department head. I helped to inform the candidate of the department’s assets as well as review what the candidate could do for the department. Reference: Gregor Kovacic, kovacg@rpi.edu.

Teaching Reviews, CIMS, 2011

I have been asked by the clinical professors at Courant to review the teaching quality of instructors for Calculus 1 on several occasions. Reference: Matthew Leingang, leingang@courant.nyu.edu.

NYC Teaching Seminar, CIMS and Columbia, Fall 2012

This is a joint effort between Columbia and New York University. The group meets once a month to discuss methods of enriching undergraduate mathematics education. We have a different topic each month, such as mathematical pedagogy, and read relevant papers for later discussion. Reference: Corrin Clarkson, clarkson@math.columbia.edu.

CIMS SURE program, Summer 2012

I helped to organize and run the Summer Undergraduate Research Experience (SURE) during the summer of 2012. I reserved group work spaces for the students and directed biweekly research status meetings where the students gave short oral reports on their progress. I advised my own research student and provided computational support and mentoring for the others. Reference: Tom Lagatta, lagatta@cims.nyu.edu.

Society for Mathematical Biology Education Group, 2012

I joined this group at the last Society for Mathematical Biology Meeting at the University of Knoxville, Tennessee. We have a Google Group where we share education resources and discuss education techniques and research. Reference: Carrie Diaz-Eaton, mathprofcarrie@gmail.com.

Collaborations

I am currently working on a paper on Changing Education in the Sciences with Alex Nectow, a graduate student of neuroscience at Rockefeller University.

4 Selected Education Talks

Calculus for the Natural and Physical Sciences
International Symposium on Biomathematics and Ecology: Education and Research, Nov. 2012

What you can do with a career in Mathematics
Capital Region Young Scholars Program, Fall 2005

5 Honors and Awards

RPI Ralph Ernest Huston Teaching Prize, 2008
Courant Institute Instructor

Undergraduate Research in Applied and Computational Mathematics
MATH-4960
Spring 2008

Instructor: Mark H. Holmes
Email: holmes (@rpi.edu)
Office Hours: TW 9:30-10:30
Office: Amos Eaton 322
Phone: 276-6891

TA: Lisa Rogers
Email: rogerl (@rpi.edu)
Office Hours: T12-2, W4-5
Office: Amos Eaton 430

Tentative Schedule

Weeks 1-4	Numerical Differential Equations, MATLAB, and Introduction to Projects
Weeks 5-6	Kinetics, and Scientific Communication
Weeks 7+	Research Seminars
Last Week	Poster Presentations

Course Web Page http://eaton.math.rpi.edu/faculty/Holmes/Courses/CSUMS/Spring08

General References (see CSUMS library for project related references)
Introduction to Numerical Solution in Differential Equations by M. H. Holmes
Introduction to Foundations to Applied Mathematics by M. H. Holmes (notes available)
Introduction to Perturbation Methods by M. H. Holmes
Mathematics Applied to Deterministic Problems in the Natural Sciences by Lin and Segel

Grading: Homework 1/4, Attendance (and Participation) 1/8, Project 5/8

Projects
Each student will be required to undertake a research-oriented project. You can either propose a project, or select one from the CSUMS library. The latter contains papers that provide background for potential project areas and these will be discussed in class. The exact schedule of events in the course is fluid, but it will be approximately as follows:
Middle of February: each student submits report on three math research papers, each from a different area
End of February: research seminar begins
End of Semester: poster presentations

Extra Events

The unique nature of this course provides opportunities for special events that will take place throughout the semester. One will be outside speakers. We will be inviting mathematicians to come to Rensselaer to talk to you about their research. They will also give regular research colloquia. When appropriate, we will ask the class to attend these lectures (and lectures of other visitors). Another event is the Hudson River Undergraduate Mathematics Conference, which takes place in April.

Miscellaneous

All computing will be done in MATLAB and this program will be introduced as the course progresses. RPI has a site license for MATLAB and you can contact the Help Desk in the VCC for more information about this. You will also need a word processor that is capable of formatting math expressions, as well as software capable of producing posters with math expressions. Examples are: MS Word and PowerPoint, and LaTeX/Keynote (the latter is what the instructor uses).

Table 1: **Mathematical Modeling Syllabus**

Day	Section	Topics	Activities
Day 1	Morning	Introduction, Go over Honor Code	Outline steps of model making, cellular growth introduction. Finite difference equations. Puzzles.
	Afternoon	Pretest. Statistics and Modeling, Economic Models	Formulate Supply and Demand Models. Inflation and compound interest. More on Finite Difference
	Evening	Economic Models	Finish up Supply and Demand models
Day 2	Morning	Algorithms and Finite difference equations.	Red blood cell growth activity. Search and Sort algorithms.
	Afternoon	Algorithms and Finite Difference Equations	Create finite difference equations based on information given. Utilizing the steps of model making.
	Evening	Conclude lesson. Introduce graph theory	Finish classwork. Read Chapter 5 and beginning of Chapter 6.
Day 3	Morning	Graph Theory	Linking graph theory to algorithms. Finish reading. Types of graphs. Euler and Hamilton.
	Afternoon	Graph Theory continued. Introduce set theory.	Read article about Paul Erdos. Cover game trees, family trees. Library for research.
	Evening	Graph Theory	Finish family Trees
Day 4	Morning	Game Day: Number Puzzles	Cover series, sequences, golden ratio, Fibonacci. Article on Golden Ratio.
	Afternoon	Benford's Law. Series and Sequences cont.	Read Ch. 8, lecture and exercises.
	Evening	Benford's Law and series	Book exercises, packet on Benford's Law.
Day 5	Morning	Fractal Geometry	Analyzing and depicting fractals and sequences found in nature. Read Chapter 12.
	Afternoon	Ideas for research. Go over Benford's Law.	Brainstorm research ideas, library trip.
	Evening		

Day 6	Morning	Fractal Geometry: African Fractals	Discuss Ron Eglash, computer lab activity on analyzing African Fractals
	Afternoon	African Fractals	Finding fractals exhibited in other communities in history
	Evening	Fractals continued	Create your own fractal construction
Day 7	Morning	Geometric modeling: Polygons	Applications and optimization discussion
	Afternoon	Polygonal optimization	Breaking simple images into polygons, exploring equations of motion
	Evening	Polygonal optimization continued	Worksheet on constructing basic graphics using polygons
Day 8	Morning	Geometric modeling: Video game applications	More on polygonal optimization, computer demonstration
	Afternoon	Geometric modeling continued	Computer lab activity
	Evening	Packet on geometric modeling in video game applications and simulations	Short essay on simulations
Day 9	Morning	Solar System Dynamics	Newton's Laws and gravitational attraction
	Afternoon	Solar System Dynamics	Depiction of forces exerted on planets
	Evening	Finish Solar System Dynamics	Finish in class activities
Day 10	Morning	Flow models: Traffic flow	Explore various scenarios of traffic flow, encorporate shortest path lesson to create models with least amount of traffic flow
	Afternoon	Flow models: Heat and fluid flow	Use of trigonometry to demonstrate flow
	Evening	Flow models continued	Examples on depiction of waves by trigonometric sequences

Day 11	Morning	Biological modeling: SIR models revisited	Exploration of types of SIR models, create a SIR model depending on varying scenarios
	Afternoon	Biological modeling continued	Ethics of biological modeling
	Evening	Reading about bacteria growth in preparation for Day 12	
Day 12	Morning	Biological growth models	Bacteria growth activity
	Afternoon	Introduction to basic chemical kinetics	Chemical oscillators discussion, exploration of examples
	Evening	Packet on REM cycle and neurological activities	Short response on importance of modeling human physiological activities
Day 13	Morning	Predator-Prey models	Rabbits vs. Wolves, exploration of varios predator-prey combination scenarios
	Afternoon	Continuation of predator-prey models, discuss presentations	Complete in class activities
	Evening	Work on final projects	
Day 14	Morning	Presentation of Final Projects	
	Afternoon	Presentation of Final Projects	
	Evening		
Day 15	Morning	Mathematical Model of the Human Sleep/Wake Cycle Presentation	Culmination of learning experience
	Afternoon		
	Evening		

V63.0123.001: Calculus III

Instructor: Lisa Rogers
Email: lrogers@cims.nyu.edu
Office: Warren Weaver Hall 921
Office Hours: Tuesday 11-12, Thursday 11-12 and by appointment

Welcome to Calc III! In this course we will take the concepts of single-variable calculus (Calc I and II) and look at their generalizations to functions of two or three variables.

Courses meet MW or TR for 110 minutes each class period. A list of sections, their meeting times and locations, and their instructors can be found on through the Registrar's Class Schedule Search. If you enter "Calculus III" in the course name search field, the current term's listings will automatically come up.

Students who wish to enroll in Calculus III must meet one of the following prerequisites:

- Calculus II (V63.0122) with a C or higher.
- Our department's Calculus III placement test.

A score of 5 on the Advanced Placement Calculus BC test does not per se qualify for enrollment in Calculus III. See the math department's placement web page for more information.

Essential Calculus, Early Transcendentals by James Stewart is the official textbook for the course. You also need access to WebAssign. NYU has a custom imprint of this text which is sold bundled with access to Enhanced WebAssign. Enhanced WebAssign includes a hyperlinked electronic format of Stewart's Calculus, Early Transcendentals (of which Essentials is a slimmed-down version) accessible through the web.

In addition to the hardcover custom textbook, the NYU bookstore also has a limited number of looseleaf printings on three-hole punched paper, bundled with access to Enhanced WebAssign. These are less expensive up front, easier to carry around (since you don't have to carry the entire textbook at once), but cannot be sold back to the bookstore.

You may also buy the latest edition of Essential Calculus, Early Transcendentals, ISBN-13 978-0-495-01428-7 non-customized, elsewhere. Then you can buy WebAssign (regular or Enhanced with Stewart's Calculus, Early Transcendentals) from them online directly.

Finally, you may decide to go completely electronic. You may buy online (through iChapters) an electronic format of Essential Calculus, Early Transcendentals, or a subset of that text consisting of which chapters you will need. This will be the exact same text, problem numbering, section numbering, and pagination as the official edition, but is not hyperlinked. With this you could buy regular WebAssign without the electronic text included. Or, you may buy Enhanced WebAssign alone, and we will provide the correspondence between problems assigned in Essentials with those from the full version of Calculus. Or, of course, you could buy both the electronic format of Essential Calculus, Early Transcendentals, and Enhanced WebAssign which includes the hyperlinked version of Essential Calculus, Early Transcendentals.

There will be weekly online assignments (around 3 per week) administered through WebAssign. WebAssign problems are computational in nature and assess the techniques introduced in class. Many of these problems will resemble examples in the textbook or from class. You will get immediate feedback on your progress and will get several chances to ensure it.

There will also be written assignments each week to turn in (approximately 15 total). These problems will require more than just procedure, might connect two more more things together, and will more closely resemble the harder exam problems.

Exams will contain a mixture of computational and conceptual problems. Some of them will resemble homework problems, while some will be brand new to you. Some component of class will also be graded. This classwork will take the form of quizzes and worksheets (approximately 1 quiz every other week, worksheets on the weeks we don't have a quiz).

Graders will grade the written homework promptly, and solutions will be made available on the course website. Graders will be expecting you to express your ideas clearly, legibly, and completely, often requiring complete English sentences rather than merely just a long string of equations or unconnected mathematical expressions. This means you could lose points for unexplained answers.

In fairness to fellow students and to graders, late homework will generally not be accepted. Because sometimes things more important than math homework come up, you have some free passes: Your TWO lowest written assignment scores and your THREE lowest WebAssign scores will be dropped in the final grade calculation. Since you have quizzes every other week (this falls under the Classwork category) your ONE lowest quiz grade will be dropped.

By all means you may work in groups on the homework assignments. Collaboration is a big part of learning and of scholarship in general. However, each student must turn in his or her own write-up of the solutions, with an acknowledgment of collaborators. There is free math tutoring sponsored by the math department, meeting in room 524 of Warren Weaver Hall. Check the signs posted throughout WWH and the tutoring web page.

If you require additional accommodations as determined by the Center for Student Disabilities, please let your instructor know **as soon as possible!**

Scheduled out-of-sequence exams and quizzes (those not arising from emergencies) must be taken before the actual exam. Otherwise, please talk to your instructor before you return to class. We may approve out-of-sequence exams in the following cases:

1. A documented medical excuse.
2. A University sponsored event such as an athletic tournament, a play, or a musical performance. Athletic practices and rehearsals do not fall into this category. Please have your coach, conductor, or other faculty advisor contact your instructor.
3. A religious holiday.
4. Extreme hardship such as a family emergency.

Your course score will be determined as the following weighted average:

Item	**Weight**
Midterm	20%
WebAssign	10%
Written Homework	15%
Classwork	15%
Final	40%
Total	100%

We will convert this score to a letter grade beginning with these values as cutoffs:

Cutoff	**Letter Grade**
93	A
90	A-
87	B+
83	B
80	B-
75	C+
65	C
50	D

These cutoffs might be adjusted, but only in the downward direction (to make letter grades higher).

A graphing calculator is encouraged for class discussion and on homework, but not allowed for exams. No specific calculator is endorsed, so do not buy a new one. If you have one already, continue to use that one; if you do not, try free alternatives such as Wolfram Alpha.

Dates	Section	Topic
1/24-28	10.1	Three Dimensional Coordinate Systems
	10.2	Vectors
1/31-2/4	10.3	The Dot Product
	10.4	The Cross Product
2/7-11	10.5	Equations of Lines and Planes
	10.6	Cylinders and Quadric Surfaces
2/14-18	10.7	Vector Functions and Space Curves
	10.8	Arc Length and Curvature
	10.9	Motion in Space: Velocity and Acceleration
2/21-25	11.1	Functions of Several Variables
	11.2	Limits and Continuity
2/28-3/4	11.3	Partial Derivatives
	11.4	Tangent Planes and Linear Approximation
	11.5	The Chain Rule
3/7-11	11.6	Directional Derivatives and the Gradient Vector
		Review
		Midterm on 3/9
3/14-18	No Classes	Spring Break
3/21-25	11.7	Maximum and Minimum Values
	11.8	Lagrange Multipliers
3/28-4/1	12.1	Double Integrals over Rectangles
	12.2	Double Integrals over General Regions
	12.3	Double Integrals in Polar Coordinates
4/4-8	12.5	Triple Integrals
	12.6	Triple Integrals in Cylindrical Coordinates
	12.7	Triple Integrals in Spherical Coordinates
4/11-15	13.1	Vector Fields
	13.2	Line Integrals
	13.3	The Fundamental Theorem of Line Integrals
4/18-22	13.4	Green's Theorem
	13.5	Curl and Divergence
4/25-29	13.6	Parametric Surfaces and their Areas
	13.7	Surface Integrals
5/2-6	13.8	Stokes' Theorem
	13.9	The Divergence Theorem
5/9		Final Review

V63.0123.001: Calculus III
Worksheet 3

1. Find a vector function that represents the curve of intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1 + y$.

2. If $\mathbf{r}(t) \neq \mathbf{0}$, show that $\frac{d}{dt}|\mathbf{r}(t)| = \frac{\mathbf{r}(t) \cdot \mathbf{r}'(t)}{|\mathbf{r}(t)|}$.

3. Reparameterize the curve $\mathbf{r}(t) = 2t\mathbf{i} + (1 - 3t)\mathbf{j} + (5 + 4t)\mathbf{k}$ with respect to arc length measured from where $t = 0$ in the direction of increasing t.

4. Find the velocity and positions vectors of a particle that has the given acceleration and the given initial velocity and position.
   $\mathbf{a}(t) = \langle t, e^t, e^{-t} \rangle \qquad \mathbf{v}(0) = \mathbf{k} \qquad \mathbf{r}(0) = \mathbf{j} + \mathbf{k}$

5. The Doctor needs to travel again! The TARDIS takes off with an initial speed of 500 meters per second and an angle of elevation 30 degrees. However, something malfunctions, only space is traversed (not time) and it makes a crash landing moments later. Find the total horizontal distance the TARDIS has traveled, the maximum height it reaches and the time it has taken to reach the ground again.

BONUS Draw the Doctor's flight path from the last question.

V63.0122.013: Calculus II, Section 13

Instructor: Lisa Rogers
Email: lrogers@cims.nyu.edu
Office: Warren Weaver Hall 921
Office Hours: Monday and Wednesday 2-3pm and by appointment

1 Objectives

Welcome to Calc II! Calculus II is a second semester calculus course for students who have previously been introduced to the basic ideas of differential and integral calculus. Over the semester, we will study three topics:

- Applications and methods of integration
- Infinite series and the representation of functions by power series
- Parametric curves in the plane.

We want you to leave the course not only with computational ability, but with the ability to use these notions in their natural scientific context, and with an appreciation of their mathematical beauty and power. By the end of the course, students should be able to do the following:

- Integrate elementary functions
- Use integration to compute area, volume, arc length, work, etc.
- Determine convergence or divergence of sequences and series
- Analyze functions expressed as a power series
- Examine curves in the plane expressed parametrically
- Communicate mathematically, including understanding, make and critiquing mathematical arguments.

2 Requirements and Prerequisites

Courses meet Tuesdays and Thursdays from 11 am to 12:50pm in Meyer 157. A list of other sections, their meeting times and locations, and their instructors can be found on through the Registrar's Class Schedule Search. If you enter "Calculus II" in the course name search field, the current term's listings will automatically come up.

Students who wish to enroll in Calculus II must meet one of the following prerequisites:

- Calculus I (V63.0121) with a C or higher.
- A score of 4 or higher on the Advanced Placement (AP) Calculus AB test
- A score of 4 or higher on the Advanced Placement Calculus BC test
- Placement into Calculus II by our departmental placement test.

See the math department's placement web page for more information.

3 Resources

Essential Calculus, Early Transcendentals by James Stewart is the official textbook for the course. You also need access to WebAssign. NYU has a custom imprint of this text which is sold bundled with access to Enhanced WebAssign. Enhanced WebAssign includes videos of worked-out problems and a hyperlinked electronic format of the text.

In addition to the hardcover custom textbook, the NYU bookstore also has a limited number of looseleaf printings on three-hole punched paper, bundled with access to Enhanced WebAssign. These are less expensive up front, but cannot be sold back to the bookstore.

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Finally, you may decide to go completely electronic with Enhanced WebAssign alone. It includes the text in a digital format.

DO NOT illegally download the text or any other copyrighted material!

A graphing calculator is encouraged for class discussion and homework, but not allowed for exams and quizzes. No specific calculator is endorsed, so do not buy a new one. You may try free alternatives such as Wolfram Alpha.

4 Assignments, Quizzes and Exams

There will be weekly online assignments administered through WebAssign. WebAssign problems are computational in nature and assess the techniques introduced in class. Many of these problems will resemble examples in the textbook or from class. You will get immediate feedback on your progress and will get several chances to ensure it. WebAssign is accessed directly through the course's Blackboard website.

There will also be written assignments each week to turn in. These problems will require more than just procedure, might connect two more more things together, and will more closely resemble the harder exam problems.

There will be graded classwork in the form of worksheets and/or quizzes every week to test your conceptual understanding of the topics learned. The worksheets will involve mostly short answer, applied problems. You will discuss and complete these worksheets in small groups and approach the worksheets as you would a laboratory writeup. Quizzes will involve shorter, more computational problems and will be completed individually.

During the semester there will be a midterm exam in class. There will also be a final exam scheduled during the Fall exam week. Exams will contain a mixture of computational and conceptual problems. Some of them will resemble homework problems, while some will be brand new to you.

Graders will grade the written homework promptly, and solutions will be made available on the course website. Graders will be expecting you to express your ideas clearly, legibly, and completely, often requiring complete English sentences rather than merely just a long string of equations or unconnected mathematical expressions. This means you could lose points for unexplained answers.

By all means you may work in groups on the homework assignments. Each student must turn in his or her own write-up of the solutions, with an acknowledgment of collaborators. There is free math tutoring sponsored by the math department, meeting in room 524 of Warren Weaver Hall. Check the signs posted throughout WWH and the tutoring web page.

If you require additional accommodations as determined by the Center for Student Disabilities, please let your instructor know **as soon as possible!**

5 Policy for Missed Classes, Assignments and Exams

In fairness to fellow students and to graders, late homework will generally not be accepted. Because sometimes things more important than math homework come up, you have some free passes: Your TWO lowest written assignment scores and your THREE lowest WebAssign scores will be dropped in the final grade calculation. Your ONE lowest classwork score (either a worksheet or a quiz) will be dropped.

Scheduled out-of-sequence exams and quizzes (those not arising from emergencies) must be taken before the actual exam. Otherwise, please talk to me before you return to class. We may approve out-of-sequence exams in the following cases:

1. A documented medical excuse.
2. A University sponsored event such as an athletic tournament, a play, or a musical performance. Athletic practices and rehearsals do not fall into this category. Please have your coach, conductor, or other faculty advisor contact your instructor.
3. A religious holiday.
4. Extreme hardship such as a family emergency.

6 Evaluation

Your course score will be determined as the following weighted average:

Item	**Weight**
Midterm	20%
WebAssign	10%
Written Homework	10%
Integration Quiz	10%
Classwork	15%
Final	35%
Total	**100%**

We will convert this score to a letter grade beginning with these values as cutoffs:

Cutoff	**Letter Grade**
93	A
90	A-
87	B+
83	B
80	B-
75	C+
65	C
50	D

These cutoffs might be adjusted, but only in the downward direction (to make letter grades higher).

7 Schedule

Dates	Section	Topic
9/4/2012 - 9/6/2012	5.1-5.5	Welcome and Integral Review
9/11/12	6.1	Integration by Parts
9/13/12	6.2	Trigonometric Integrals and Substitution
9/18/12	6.3	Partial Fraction Decompositions
9/20/12	6.5	Approximate Integration
9/25/12	6.6	Improper Integrals
9/27/12	7.1	Areas between Curves
10/2/12		**Integration Quiz**
10/4/12	7.2	Volumes
10/9/12	7.3	Volumes by Cylindrical Shells
10/11/12	7.4	Arc Length and Introduction to Parameterization
10/16/12		**Fall Break - No Classes**
10/18/12	7.6	Introduction to Differential Equations
10/23/12		**Midterm Exam**
10/25/12	8.1	Introduction to Sequences
10/30/12	8.2	Introduction to Series
11/1/12	8.3	Integral and Comparison Tests
11/6/12	8.4	Alternating Series and other Convergence Tests
11/8/12	8.5	The Power Series
11/13/12	8.6	Representing Functions as Power Series
11/15/12	8.7	Taylor and Maclaurin Series
11/20/12	8.8	Applications of Taylor Polynomials
11/22/12 - 11/25/12		**Thanksgiving Recess - No class**
11/27/12	9.1	Parametric Curves
11/29/12	9.2	Calculus with Parametric Curves
12/4/12	9.3	Polar Coordinates
12/6/12	9.4	Areas and Lengths in Polar Coordinates
12/11/12		Review Day/ Catch-up Day
12/13/12		Review Day/Catch-up Day
12/17/12		**Final Exam**

V63.0122.013: Calculus II
Worksheet 1

Name:

1 Escape Velocity

A classic problem from physics is the study of an object shot away from a planet that is acted upon only by gravitational forces. We would like to study the velocity of this object as it moves away from the surface of a planet using Newton's laws of gravitational attraction and motion, but ignoring air resistance. Newton's law of gravitational attraction says the following: The force of attraction between two objects is inversely proportional to the square of the distance between the objects. In equation form, that is

$$F_1 = G\frac{m_1 m_2}{x^2} = F_2$$

where F_1 is the force generated by the object with mass m_1, F_2 is the force generated by the object with mass m_2, x is the distance between the centers (of gravity) of the objects, and G is the gravitational constant. The Earth, however, is a massive object that generates an equally massive gravitational field. Thus, we must calculate the velocity in terms of Earth's gravitational field. We do this by first modifying Newton's Law of Gravitational Attraction to represent the the force generated by Earth's gravitational field, which is

$$F = -G\frac{mR^2}{(x+R)^2}$$

where R is the radius of the Earth. To find the velocity of the projectile, we equate F with its definition from the laws of motion, $F = ma$, where a is the acceleration of the projectile and m is the mass of the projectile. We then have the equation

$$ma = -G\frac{mR^2}{(x+R)^2}$$

Note that x is a function of t.[1]

[1]Problem courtesy of Joe Mahaffy

1. Use the chain rule to write the acceleration of the projectile, a in terms of velocity, v.

2. We now have a **differential equation** describing the velocity as a function of the distance from the surface of the Earth. We can now divide by the appropriate quantities and get a term on the left side that is a function of one variable, and a term on the right side that is a function of another variable. What is the new form of the equation?

3. We can now integrate both sides. Use an appropriate substitution to simplify one of the integrals. Solve the resulting equation for velocity in terms of position, x.

4. If $v(0) = V_0$, what is the velocity in terms of position and V_0?

5. The **escape velocity** is the velocity at the surface of the planet, V_0, required for an object to escape the gravitational pull of the Earth and not return. Velocity of the object approaches zero as position, x,

increases. In terms of g and R, what is the escape velocity of an object?

2 Dead Space in the Lungs

When breathing air into and out of the lungs, the air must pass through the nasal passageways, the pharynx, the trachea, and the bronchi before it can enter the alveoli where the oxygen and carbon dioxide exchange with the circulatory system. These regions where vital gasses cannot be exchanged are called **dead spaces**. To determine the health of patients with respiratory problems, it is important to know information on all aspects of the lungs, including the measurement of the dead space.

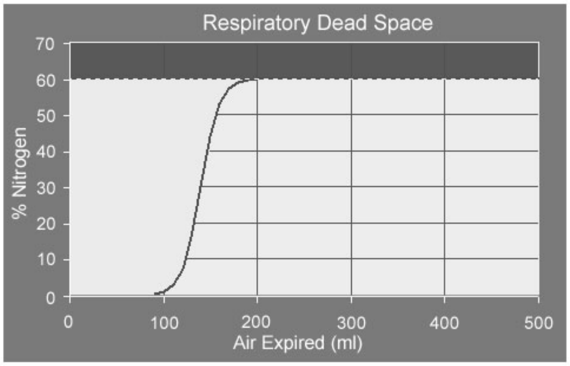
There is a fairly simple means of measuring the dead space for a patient. The patient breathes normal air, then just before the measurement, he or she takes a breath of pure oxygen. The oxygen will mix with the normal air in the alveoli, but the dead space will be filled almost exclusively with pure oxygen. The patient expires the mixture through a rapidly recording nitrogen meter. The recording gives a measurement of the amount of nitrogen. The part that only includes oxygen represents the dead space. Below is a graph showing a typical recording of a patient.[2]

The region to the left of the curve is the pure oxygen in the dead space, while the region to the right of the curve represents the mixed air in the alveoli, where the actual gas is exchanged with the circulatory system. The volume of the dead space is given by the area to the left of the curve times the total volume of air expired divided by the total area under the dotted line.

A function that closely approximates the data collected and displayed above is given by

$$N(x) = 0.3 + 0.3\frac{e^{0.05(x-140)} - e^{-0.05(x-140)}}{e^{0.05(x-140)} + e^{-0.05(x-140)}}$$

[2] Problem courtesy of Joe Mahaffy



where N is the percent of nitrogen in the expired air and x is the number of milliliters expired. We can readily see that the total area under the dotted line, A, is $A = 0.6 * 500 = 300ml$. You will use this information to answer the following questions.

1. In words, how would we find the area to the left of the curve $N(x)$?

2. What is the definite integral for finding the area to the left of $N(x)$?

3. What substitution must we make to simplify and solve this integral?

4. Compute the integral above.

5. What is the volume of the dead space in the above graph?

6. Breathing is cyclic, and a full respiratory cycle from the beginning of the inhalation to the end of exhalation takes about 5 seconds. The maximum rate of air flow into the lungs is about 0.5 L/s. A reasonable model for the rate of airflow is given by the function $f(t) = \frac{1}{2}\sin(\frac{2\pi t}{5})$.

   (a) Why is $f(t) = \frac{1}{2}\sin(\frac{2\pi t}{5})$ a reasonable model for airflow rate?

   (b) Use this model to find the volume of inhaled air in the lungs at time t.

Calculus for the Natural and Physical Sciences

Lisa Rogers

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BEER 2012

Outline

- Course Development
  - Motivation
  - Objectives
  - Course Structure
  - Resources
  - Key Components
  - Topics and Applications
  - Classwork
  - Project
- Previous Experiences
- Calculus at CIMS

Motivation

- Increase in interdisciplinary interaction
- Emergence of new fields of research with little change to structure of core quantitative courses
- Teaching to optimize learning and memory

Objectives

- Mastery of core standards of Calculus I and II
- Development of logical problem solving strategies, including analysis of one's own and others' methods and solutions (creative and critical thinking)
- Development of interdisciplinary oral and written communication skills
- Development of quantitative reasoning skills

Course Structure

- No prerequisites - first year Calculus course
- Two semesters - Calculus I and II material
- Lecture/Problem Solving/Discussion Period
- Computer Lab Period

Resources

- Stewart Calculus for a guide
- COMAP Tools Modules
- Applied Math Problems Group
- Mathematics for the Life Sciences
- Joe Mahaffy's Math 121 and 122 Webpages
- Wolfram Demonstrations Project

Key Components

- Lecture: less of it!
- Visualizations: Wolfram, Illustrative Math Project
- Problem Based Learning
- Group Work
- Hands-on computational component
- Scientific Writing comprehension
- Instructor Interaction
- Graduate Students

Topics: Mathematical Concepts

- Calculus I
  - Thinking geometrically: Visualizing functions, coordinate systems
  - Limits and Continuity
  - Rates if change and the derivative
  - Methods of Differentiation
  - Related rates: Introduction to formulating a mathematical model
  - Linearization
  - Optimization

- Calculus II
  - The relationship between Derivatives and Antiderivatives
  - The Integral
  - Integration Techniques
  - Intro to Differential Equations

Topics: Biological/ Physical Concepts and their Mathematical Relations

Introduce biological/physical ideas with mathematical consequences

- Newton's Laws: Motion and Gravitational Pull. Tangent lines, substitution, separable equations, initial value problems.
- Chemical Kinetics: Limits and Equilibria, Differential equations
- Cardiac Cycle: Dimensional Analysis, Volume Control
- Growth of Populations: Allometric, Malthusian, Logistic. Difference Equations, Partial Fractions, Substitution, Error and Sensitivity Analysis
- Ultradian and Circadian rhythms: Optimization, Trigonometric functions

Classwork

- Computational component: Matlab, Octave
- Written component: LaTex, by hand
- Oral component: Group work, presentation

Project - Second Semester

- Bank of self-contained research papers
- Oral and written report explaining model to peers

Sample Projects

Immunological and Epidemiological HIV/AIDS Modeling, Isihara 2006. UMAP Modules 2006: Tools for Teaching.

1. Introduction

HIV/AIDS is arguably the number one epidemic today. The United Nations organization UNAIDS [2004] reported these staggering estimates for 2004:

- 39.4 million people have HIV/AIDS;
- 2.3 million children under the age of 15 have HIV/AIDS;
- 4.9 million new cases of HIV were reported;
- 640,000 children under 15 were newly infected with HIV (the vast majority from their mothers); and
- over 3 million people died of AIDS (an average of more than 8,000 per day).

3. Background in Differential Equations

In the differential equations that we consider in this Module, the independent variable t represents time, and that is the only variable that we differentiate with respect to. Because partial derivatives are not involved, our differential equations are called *ordinary* differential equations (ODEs). We begin this brief tutorial on ODEs by discussing several important *scalar equations*, i.e., equations involving a single function of time t. We then proceed to *ODE systems*, which involve two or more functions of t. (This section can be skipped by those conversant with ordinary differential equations including stability analysis of equilibria in nonlinear systems.)

3.1 Exponential and Logistic Growth

3.1.1 Exponential Growth

In modeling the dynamics of some population $x = x(t)$, it may be reasonable to assume that the rate of increase in population is proportional to the size of the population. In this case,

$$x' = kx,$$

Do males matter? The role of males in population dynamics, Rankin D and Kokko H. Oikos 116: 335 - 348, 2007.

Table 1. Generalised effects of removing males on overall population density.

The effect of removing males on population density	
Density increases	Density decreases
Density dependence	Sperm limitation
Higher male resource requirement	Parental care
Sexual segregation	Sexual segregation
Sexual harassment	Male nuptial gifts
Disease transmission	Infanticide

Do Males Matter?

$$\frac{dM}{dt} = f(F, M)brF - g_M(F, M)M$$

$$\frac{dF}{dt} = f(F, M)b(1 - r)F - g_F(F, M)F$$

Previous Experiences

- RPI: Project CSUMS
- Mathematical Contest in Modeling

Project CSUMS

NSF funded program in Computational Science Training for Undergraduates in the Mathematical Sciences.

- Goal: To better prepare undergraduate mathematics students to pursue careers and graduate study in fields that require integrated strengths in computation and mathematical sciences.
- Layout: Several weeks of lecture-based training in computational and mathematical techniques followed by mentored research meetings of student group and individual projects. Includes regular lectures by professional research scientists on their topic of specialty.
- Culminates with oral and written presentations of student projects. Exceptional groups of students receive funding to continue mentored research during the summer months.

Sample Projects

- The Maillard Reaction - Chemical Kinetics of Baking
- Creating an Invisibility Cloak - Negative Index of Refraction
- Effects of Acid Rain on Lake Ecosystems - Population Dynamics
- Low Reynolds Number Swimmers

Mathematical Contest in Modeling

- Students work in small groups over the course of 4 days to produce a paper addressing and solving a current applied math problem
- Students of all levels and backgrounds
- Practice problem sessions to develop skills

How we practice

- Review of a previous MCM problem
- Brief lecture on mathematical concepts involved
- Group attack of problem:
  - Realistic Physical Assumptions, simplifying assumptions
  - Variables, Parameters
  - Mathematical Concepts
  - Algorithmic Approach
  - Pros and Cons of Model

Designing a Traffic Circle: Continuous

Many cities and communities have traffic circles from large ones with many lanes in the circle, to small ones with one or two lanes in the circle. The goal of this problem is to use a model to determine how best to control traffic flow in, around, and out of a circle. State clearly the objective(s) you use in your model for making the optimal choice as well as the factors that affect this choice. Include a Technical Summary of not more than two double-spaced pages that explains to a Traffic Engineer how to use your model to help choose the appropriate flow-control method for any specific traffic circle. That is, summarize the conditions under which each type of traffic-control method should be used. When traffic lights are recommended, explain a method for determining how many seconds each light should remain green (which may vary according to the time of day and other factors). Illustrate how your model works with specific examples.

Gamma Knife Treatment Planning: Discrete

The gamma knife unit delivers a single high dose of ionizing radiation to a radiographically well-defined, small intracranial 3D brain tumor without delivering any significant fraction of the prescribed dose to the surrounding brain tissue. Many beams simultaneously intersect at the isocenter, resulting in a spherical dose distribution at the effective dose levels. Irradiating the isocenter to deliver dose is termed a “shot." Shots can be represented as different spheres. There are various beams for irradiating different size volumes. For a target volume larger than one shot, multiple shots can be used to cover the entire target. In practice, most target volumes are treated with 1 to 15 shots. The target volume is a bounded, three-dimensional digital image that usually consists of millions of points. In general, an optimal treatment plan is designed to meet several requirements (given) with certain constraints (given).

Calculus at CIMS

- Calculus II
- Lectures
- Problem Based Learning
- Lab-like Worksheets

For a continuous random variable x, we have a **probability density function** $f(x)$ such that

1. $f(x) \geq 0$ for all x
2. $\int_{-\infty}^{\infty} f(x)dx = 1$
3. $P(a \leq x \leq b) = \int_a^b f(x)dx$ where $P(a \leq x \leq b)$ is the probability that the random variable x is in the interval $[a, b]$.

The **mean** of a probability density function is then defined as

$$\mu = \int_{-\infty}^{\infty} xf(x)dx$$

and the **standard deviation** as

$$\sigma = \left(\int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx \right)^{\frac{1}{2}}$$

The best fit to this data is the exponential decay function $S(x) = 502e^{-2.08x}$. After normalization (division by a quantity so the integral over the entire range of the probabilty density is one), we find that the continuous probability density function that best describes $S(x)$ is $p(x) = ke^{-kx}$. The probability of finding a seed in an interval $[x_1, x_2]$ is then $\int_{x_1}^{x_2} p(x)dx$.

1. What is the probability of finding a seed between 1 and 2 meters from the plant?
2. Consider the integral $\int xe^{-2x}$. Find the solution to this using integration by parts.
3. Using integration by parts, find the solution to $\int xp(x)dx$.
4. Now solve $\int_0^a xp(x)dx$. When you find your solution in terms of integer a, take the limit of that expression as a approaches infinity. This is the mean distance that a seed travels for California Buckwheat.

A normal blood pressure for humans is 120/80. These numbers represent the force on the arterial walls generated by the beating of the heart. We can derive a model for **arterial pressure** at time t, $P_a(t)$. The **cardiac output**, Q, represents the average amount of blood pumped by the heart (liters per minute). The **stroke volume**, V, is the amount of blood pumped by the heart during one beat (liters per beat). This volume is the amount that resides in the aorta at the beginning of **systole**, the period during which the heart contracts and forces blood out of its vessels. This is then followed by a period of relaxing and refilling, or **diastole.** T is the duration of a heart beat (minutes per beat). The maximum arterial pressure occurs during systole, and so will be called P_{sys}, while the minimal arterial pressure occurs during diastole, and so will be called P_{dia}.

1. What is the cardiac output in terms of stroke volume and duration of heart beat?
2. What is $\frac{1}{T}$?

$$Q_s(t) = \frac{P_a(t) - P_v(t)}{R_s}.$$

$$P_v(t) \approx 0$$

$$V_a(t) = C_a P_a(t)$$

We assume compliance and resistance are both constant quantities. During systole the aortic valve is closed, meaning no blood flows into the aorta.

1. Write a differential equation describing the rate of change of arterial volume in terms of flow rate in and flow rate out.
2. Convert the flow rate(s) and volume to pressures. What is the new differential equation in terms of arterial pressure?
3. If $P_a(0) = P_{sys}$, what is the solution to this differential equation?

We can now estimate realistic values for parameters C_a and R_a using the solution to the differential equation for arterial pressure we just determined and experimental data.

$$V = V_{sys} - V_{dia}.$$

1. What is the stroke volume in terms of systolic and diastolic pressure?
2. Using $V = QT$, find the equation for arterial compliance, C_a.
3. We know that diastolic pressure occurs just before the next heartbeat. How would you represent this using the solution to the differential equation for arterial pressure?
4. For a normal human, we have a pulse of approximately 70 beats per minute, a cardiac output of 5.6 liters per minute, systolic pressure of $P_{sys} = 120$ millimeters of mercury and diastolic pressure of $P_{dia} = 80$ millimeters of mercury. Compute the arterial compliance and resistance for a normal human.

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Thank you!
Questions?

Email me: lrogers@cims.nyu.edu

References

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- Epp, Susanna, 1999. "The Language of Quantification in Mathematics Instruction." Excerpt from *Developing Mathematical Reasoning in Grades K-12*.
- Kopell et al, 2002. "BIO2010: Transforming Undergraduate Education for Future Research Biologists." The National Academies.

... many more. Ask for a comprehensive list!