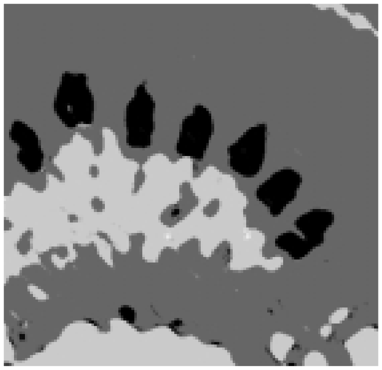

INTERDISCIPLINARY LIVELY APPLICATIONS PROJECT

The Shuttle Problem

MATERIALS

1. Problem Statement (3 Situations); Student
2. Sample Solution; Instructor
3. Notes for the instructor

Computing Requirements:
A computer algebra system such as Maple, *Mathcad*, or *Mathematica*

INTERDISCIPLINARY LIVELY APPLICATION PROJECT

TITLE: THE SHUTTLE PROBLEM

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MATHEMATICAL CLASSIFICATION: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS
DISCIPLINARY CLASSIFICATION: PHYSICS

PREREQUISITE SKILLS:
- SOLVING ORDINARY DIFFERENTIAL EQUATIONS USING THE METHOD OF SEPARATION OF VARIABLES
- DIFFERENTIATING VECTOR FUNCTIONS
- MODELING TORQUE USING THE CROSS PRODUCT

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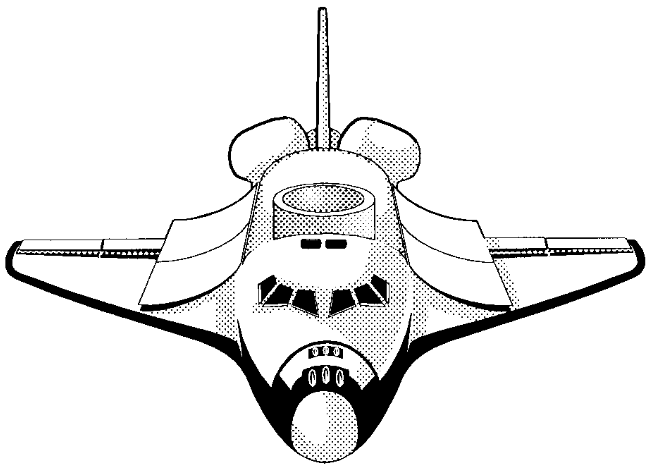
NSF INITIATIVE:
MATHEMATICS SCIENCES AND THEIR APPLICATIONS THROUGHOUT THE CURRICULUM
(CCD-MATH)

INTRODUCTION

NASA wishes to retrieve and repair a damaged satellite which is in danger of re-entering the Earth's atmosphere. NASA has called upon you to assist in various aspects of this upcoming shuttle mission. Not only will you be required to assist in the initial launch, but you will also be required to operate the robotic arm (Canadarm) in the retrieval process.



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PART 1: ESCAPE SPEED

One of the first considerations for launching the shuttle is escape speed. This is the minimum initial speed at which a projectile must be launched in order to escape the Earth's gravitational field. For the shuttle, this is an upper bound for its launch speed as we do not want the shuttle to leave Earth's gravitational field. Since it is rather complicated to incorporate the changing rates of propulsion and thrust, in this initial analysis we treat the shuttle as a projectile — an object that is initially thrown, then has no inflight propulsion.

Newton's universal law of gravitation applied to this situation is given by $F = -G\frac{M_e m_s}{r^2}$. Here, F represents the gravitational force exerted on the shuttle by the Earth, $G = 6.67 \times 10^{-11} N \frac{\text{m}^2}{\text{kg}^2}$ is a gravitational constant, $M_e = 5.98 \times 10^{24}\,\text{kg}$ is the mass of the Earth, m_s, is the mass of the shuttle (the shuttle weighs 220,000 pounds), and r is the distance from the shuttle to the center of the Earth. Additionally, the radius of the Earth is approximately 6.38×10^8 meters.

A. Starting with Newton's Second Law of Motion, derive the shuttle's escape speed. (Hint: If the shuttle launches exactly at its escape speed, then in theory, when the velocity of the shuttle slows downs to zero, it has achieved infinite distance from the Earth. (You will also need to use the chain rule in order to determine that the derivative of the velocity, v, at any time t is given by $\frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = \frac{dv}{dr}v$.

B. Comment on the feasibility of launching the shuttle at this speed; i.e., explain why it is or is not feasible, and what can be done if it is not feasible.

C. Now consider a more realistic scenario in which booster rockets are attached to the shuttle. If the booster rockets cease burning when the shuttle is 6000 miles above the Earth and the shuttle's speed at that time is 15,000 miles per hour, determine if the shuttle will escape the Earth's gravitational field.

D. Qualitatively, how does the mass of the shuttle affect the escape speed and the ability to get the shuttle into orbit? What are the implications of the shuttle burning fuel during flight?

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PART 2: ORBIT ANALYSIS

NASA wants to know the acceleration experienced by the damaged satellite as it maintains a circular orbit. The satellite's position at any point (x,y) in the plane of the orbit can be modeled by the position vector $\overline{R}(t) = x(t)\hat{i} + y(t)\hat{j}$. However, since distances are measured relative to the center of the Earth and the orbit is circular, the use of polar coordinates simplifies computations. Thus, position will be measured in terms of angular and radial components.

A. Convert the position vector to its corresponding polar form. Note that the radius, $r(t)$, and angle $\theta(t)$, are functions of time.

B. Determine the associated acceleration vector.

C. Compute the acceleration in each direction (angular and radial) and write the acceleration vector as the sum of a radial and angular component. That is, write the acceleration vector you just computed in the form $\vec{a} = a_r \vec{u}_r + a_\theta \vec{u}_\theta$, where the unit radial vector is given by $\vec{u}_r = \cos\theta\hat{i} + \sin\theta\hat{j}$,

   and the unit tangential (angular) vector is given by $\vec{u}_\theta = -\sin\theta\hat{i} + \cos\theta\hat{j}$.

   Hint: use the geometrical relationship between the two vectors $\vec{u}_r$ and $\vec{u}_\theta$.

D. Determine the acceleration vector for a satellite moving in a circular orbit 200 miles above the Earth with a constant angular speed of 4.1440 radians per hour. Interpret your results.

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PART3: SATELLITE RETRIEVAL

Due to atmospheric drag, the satellite is losing altitude so quickly that it will burn up in the upper atmosphere unless it can be immediately placed into a higher orbit. NASA originally intended to place the satellite inside the shuttle bay and move it to a higher orbit. However, due to an onboard malfunction, most of the shuttle's breathable air has been lost. There is not enough time to retrieve the satellite, repair it, and then move it. NASA feels it is far too dangerous to have astronauts outside working on the satellite while the shuttle is maneuvering, so they plan to grab the satellite with the arm and move to a higher orbit while the satellite is being brought inside the bay. The manuever to a higher orbit requires a constant firing of the shuttle's thrusters, which gives the shuttle a constant acceleration of 0.3 m/sec^2, directed away from the Earth.

You are required to operate the robotic arm to retrieve the hapless satellite. From NASA's design specifications, the hinge that connects the robotic arm to the shuttle can only support a torque of 800*N*m. The arm itself consists of a long and a short portion, connected by a joint. There is a grappling hook at the end of the short portion that can be used to snag objects, such as the satellite, from orbit (ignore its length for this analysis). The long portion of the arm has length 9 m and mass 120 kg, and the short portion of the arm has length 7 m and mass 90 kg. The satellite weighs 500 pounds on Earth.

The satellite can be brought into the cargo bay when the two portions of the arm are vertical. It is your job to retrieve the satellite without allowing mechanical failure of the joint, i.e., without surpassing the torque restriction stated above. In a recent attempted docking maneuver, the MIR space station was damaged when a supply ship collided with it. In order to avoid a similar fiasco, NASA wants the shuttle to be as far away from the satellite as possible when it is retrieved.

There are two degrees of freedom to consider; the angle between the arm and the shuttle bay and the angle between the two portions of the arm. To simplify matters, model the angles of the arm as shown in the diagram below. Assume initially that the arm is geared in such a way that angles a_1 and a_2 are the same at all times. Since the material in the arm is uniform (same type of material, density, etc.) it is also reasonable to assume that the force exerted along the length of each portion of the arm is concentrated at the center of the respective portion.

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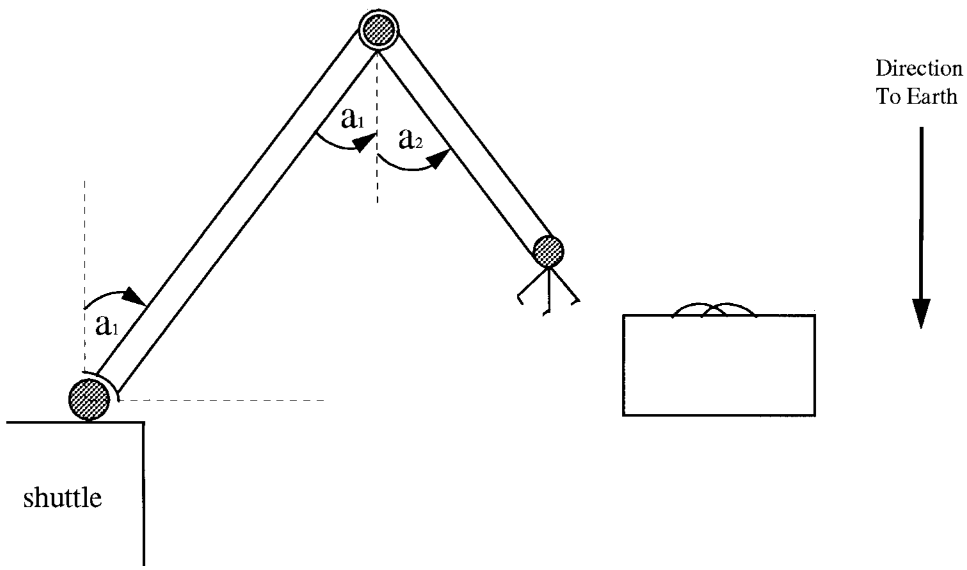

A. What is the maximum distance the satellite can be from the shuttle yet still be retrieved?

B. Assuming that the shuttle can be moved to this distance, plot the return path of the satellite from the retrieval point to the cargo bay. Be sure to describe your reference system and any additional assumptions you make.

C. To make matters more realistic, now assume that the angles a_1 and a_2 are independent of each other. Derive an equation which expresses the magnitude of the torque as a function of the angles a_1 and a_2. Then, using a contour plot, graph the magnitude of the torque against the angles a_1 and a_2. Describe any overall trends that you observe .

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