MATH-UA.251.001: Introduction to Mathematical Modeling
Homework 6

More HIV/AIDS Modeling

1. Read and complete exercises 6 and 7 from "HIV/AIDS Modeling Part 2". Read the section on chemotherapy treatment but do not actually implement exercise 8.

Lotka-Volterra Systems - Derivation and Analysis

Here we will discuss a class of problems used to describe interactions between consumers and resources, whether they be animals, humans, etc. Let's say we have 2 species that are competing for a food source: specifically Rabbits and Sheep competing for grass in a verdant field. We are not going to keep track of the rate of grass change/depletion - assume a finite supply of grass. We want to determine how the populations of both species evolve in time and with respect to one another. We have the following assumptions to satisfy:

- Each species will grow to its carrying capacity in the absence of the other species. We should assume logistic growth for both species in the absence of the other.
- Rabbits have a legendary ability to reproduce, so their intrinsic growth rate should be higher than than of the sheep.
- When rabbits and sheep encounter each other, trouble starts a-brewin'. Sometimes the rabbits get to eat, but more usually the sheep will gently nudge the rabbits out of the way and start nibbling (the grass, not the rabbits.... hopefully). Assume the conflicts occur at a rate proportional to the size of each population. For example, for twice as many sheep, the probability of a rabbit encountering a sheep will be twice as great.
- Conflicts reduce the ability of the species to consume grass, which reduces the growth rate of the species. The effects should be more severe for the rabbits because most of the time, they are the ones being

nudged out of the way so the sheep can eat. Unless of course they are super-rabbits.

With these assumptions, complete the following:

1. Write a system of differential equations representing the evolution of rabbit and sheep populations over time that incorporates the above assumptions. Use parameters to represent rates and proportionality, and show how the parameters should relate to each other (i.e. is one comparatively larger than another?). Pick comparatively appropriate values for these parameters to test.

2. Find the fixed points (or steady states) of this system. What physical state does each fixed point represent?

3. Compute the stability of each of these fixed points. For each fixed point and corresponding eigenvalue, graphically depict (by hand, not with MATLAB or any other software) the flow around the point in the Sheep-Rabbits plane.

4. Combine all of your pictures for stability for each fixed point in one graph to get a view of the complete phase plane. What does this phase plane say qualitatively about the evolution of the population of both species?